

## Arithmetic Sequences

Definition of an arithmetic sequence: A sequence is said to be arithmetic if each term, after the first, is obtained from the preceding term by adding a common value.

Examples of arithmetic sequence:

2, 4, 6, 8, 10...

-2, -4, -6, -8, -10...

$\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$ , 1,  $1\frac{1}{4}$ ...

-2, -1, 0, 1, 2...

Find the general term of an arithmetic sequence. "The  $n$ th term of an arithmetic sequence is  $a_n = a_1 + (n - 1)d$ , where  $a_1$  is the first term and  $d$  is the common difference."

Derivation of arithmetic sequence:

$a_1$

$a_2 = a_1 + d$

$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d$

$a_4 = a_3 + d = (a_1 + 2d) + d = a_1 + 3d$

Examples:

1. Find the  $n$ th term of the arithmetic sequence 11, 2, -7...
2. The first term of an arithmetic sequence is -15 and the fifth term is 13. Find the fortieth term.

Solution to Examples:

$$\begin{aligned} 1. \quad a_n &= a_1 + (n - 1)d \\ &= 11 + (n - 1)(-9) \\ &= -9n + 20 \end{aligned}$$

$$\begin{aligned} 2. \quad a_5 &= a_1 + (5 - 1)d \\ 13 &= -15 + 4d \\ 28 &= 4d \\ d &= 7 \end{aligned}$$

$$a_{40} = -15 + (39)(7) = 258$$

Definition of the sum of an arithmetic sequence:  $S_n = n/2 [2a_1 + (n - 1)d]$ .

Examples:

1. Find  $S_{20}$  for the arithmetic sequence whose first term is  $a_1 = 3$  and whose common difference is  $d = 5$ .
2. Find the sum of the first 10,000 terms of the arithmetic sequence beginning with 246, 261, and 276...

Solution to Examples:

$$\begin{aligned} 1. \quad S_{20} &= 20/2 [2(3) + (20 - 1)5] \\ &= 1010 \end{aligned}$$

$$\begin{aligned} 2. S_{10,000} &= 10,000/2 [2(246) + (10,000 - 1)15]. \\ &= 752,385,000 \end{aligned}$$

Now solve the problem stated at the beginning of the class.

## Sample Worksheet

- Which of the following is/are example(s) of arithmetic sequence?
  - 1, 3, 5, 7, ...
  - 2, 4, 8, 16, ...
  - 3, -2, -1, 0, ...
  - 1, 2, 3, 5, ...
- Find the 30<sup>th</sup> term of the sequence 120, 126, 132, 138, ...
- The first multiple of 3 after 4 is  $6 = a_1$  and the one preceding 262 is  $261 = a_n$ . Now find  $n$ .
- Find  $S_{30}$  for the sequence  $a_k = \frac{1}{2}k + 10$
- Suppose you save \$10 one week and that each week thereafter you save \$0.50 more than the preceding week. How much will you have saved by the end of 1 year?
- An editor takes a position at \$7,400 a year. She receives annual increases in salary of \$350. What will her salary be during her twelfth year of service?
- A student taking a test consisting of ten questions is told that each question after the first is worth two credits more than the preceding question. If the third question is worth fifteen credits, how much is the last question worth?

## Solution Guide to Sample Worksheet

1. A and C are examples of arithmetic sequence.
2.  $a_{30} = 120 + (30 - 1)6$   
 $= 294$
3.  $261 = 6 + (n - 1)3$   
 $n - 1 = 85$   
 $n = 86$
4.  $S_n = n/2 (a_1 + a_{80})$   
 $= 80/2 (21/2 + 50)$   
 $= 2420$
5. \$1,183
6. \$11,250
7. 29 credits