

Mixed Expressions and Complex Fractions Examples

1. Algebraic expressions such as $a + \frac{b}{c}$, and $5 + \frac{x-y}{x+3}$ are called “**mixed expressions.**” Changing mixed expressions to rational expressions is similar to changing mixed numbers to improper fractions.

Mixed expressions contain monomials and algebraic fractions.

**Mixed Number
to
Improper Fraction**

$$3\frac{2}{5} \text{ or } 3 + \frac{2}{5} = \frac{3(5)+2}{5} = \frac{15+2}{5} = \frac{17}{5}$$

**Mixed Expressions
to
Rational Expression**

$$a + \frac{a^2+b}{a-b} = \frac{a(a-b) + (a^2+b)}{a-b} = \frac{a^2-ab+a^2+b}{a-b} = \frac{2a^2-ab+b}{a-b}$$

Help students to see that they are changing numbers with unlike denominators. For example, consider $a + \frac{a^2+b}{a-b}$ as $\frac{a}{1} + \frac{a^2+b}{a-b}$. Changing the mixed expression to an algebraic fraction requires that $\frac{a}{1}$ be renamed with the LCD ($a-b$) as its denominator.



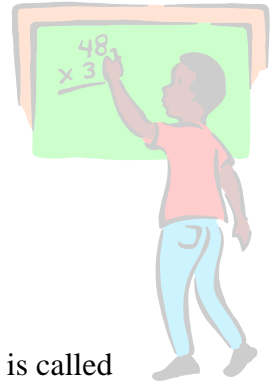
2. **Example:** Find $\rightarrow 8 + \frac{x^2 - y^2}{x^2 + y^2}$ and simplify.

Notice that $8 + \frac{x^2 - y^2}{x^2 + y^2}$ is a mixed expression. Therefore, use the method shown above to find the sum.

$$8 + \frac{x^2 - y^2}{x^2 + y^2} = \frac{8(x^2 + y^2) + (x^2 - y^2)}{x^2 + y^2} = \frac{8x^2 + 8y^2 + x^2 - y^2}{x^2 + y^2} = \frac{9x^2 + 7y^2}{x^2 + y^2}$$

3. **Example:** Find $\rightarrow m + \frac{3}{m+4}$ and simplify.

$$m + \frac{3}{m+4} = \frac{m(m+4) + 3}{m+4} = \frac{m^2 + 4m + 3}{m+4}$$



4. If a fraction has one or more fractions in the numerator or denominator, it is called a “**complex fraction.**” Some complex fractions are shown below:

$$\begin{array}{cccc} \frac{3\frac{1}{2}}{\frac{2}{3}} & \frac{8}{a} & \frac{a+b}{a-b} & \frac{1-\frac{1}{x}}{\frac{1}{x}+\frac{1}{y}} \end{array}$$



5. Consider the complex fraction $\frac{\frac{3}{5}}{\frac{7}{8}}$. To simplify this fraction rewrite it as $\frac{3}{5} \div \frac{7}{8}$

and proceed as follows.

$$\frac{3}{5} \div \frac{7}{8} \rightarrow \frac{3}{5} \cdot \frac{8}{7} \rightarrow \frac{24}{35}$$

Recall that to find the quotient, you multiply by $\frac{8}{7}$, the reciprocal of $\frac{7}{8}$

6. Similarly, to simplify $\frac{\frac{a}{b}}{\frac{c}{d}}$, rewrite it as $\frac{a}{b} \div \frac{c}{d}$ and proceed as follows.

$$\frac{a}{b} \div \frac{c}{d} \rightarrow \frac{a}{b} \cdot \frac{d}{c} \rightarrow \frac{ad}{bc}$$

When simplifying complex fractions, the following shortcut may be applied

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc} = \frac{\text{outside} \cdot \text{product}}{\text{inside} \cdot \text{product}}$$

7. Simplifying Complex Fraction Rule:

Any complex fraction $\frac{\frac{a}{b}}{\frac{c}{d}}$, where $b \neq 0$, $c \neq 0$, and $d \neq 0$, may be expressed as $\frac{ad}{bc}$.

8. **Example:** simplify $\frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} - \frac{1}{y}}$.

$$\frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} - \frac{1}{y}} \rightarrow \frac{\frac{y}{xy} + \frac{x}{xy}}{\frac{y}{xy} - \frac{x}{xy}} \rightarrow \frac{\frac{y+x}{xy}}{\frac{y-x}{xy}} \rightarrow \frac{y+x}{xy} \cdot \frac{xy}{y-x} \rightarrow \frac{y+x}{\cancel{xy}} \cdot \frac{\cancel{xy}}{y-x} \rightarrow \frac{y+x}{y-x}$$

9. **Example:** simplify $\rightarrow \frac{x+4 - \frac{1}{x+4}}{x+11 + \frac{48}{x-3}}$

$$\frac{x+4 - \frac{1}{x+4}}{x+11 + \frac{48}{x-3}} \rightarrow \frac{\frac{(x+4)(x+4) - 1}{x+4}}{\frac{(x+11)(x-3) + 48}{x-3}} \rightarrow \frac{\frac{x^2 + 8x + 16 - 1}{x+4}}{\frac{x^2 + 8x - 33 + 48}{x-3}} \rightarrow \frac{\frac{x^2 + 8x + 15}{x+4}}{\frac{x^2 + 8x + 15}{x-3}}$$

$$\frac{\overset{1}{x^2 + 8x + 15}}{x+4} \cdot \frac{x-3}{\underset{1}{x^2 + 8x + 15}} \rightarrow \frac{x-3}{x+4}$$

10. All of these calculations may seem confusing to some students. Have students separate these into two problems: one involving the numerator and one the denominator. Then put the two together.

11. **Example:** simplify $\rightarrow \frac{x - \frac{x+4}{x+1}}{x-2}$

$$\frac{x - \frac{x+4}{x+1}}{x-2} \rightarrow \frac{\frac{x(x+1) - (x+4)}{x+1}}{x-2} \rightarrow \frac{\frac{x^2 + x - x - 4}{x+1}}{x-2} \rightarrow \frac{\frac{x^2 - 4}{x+1}}{\frac{x-2}{1}} \rightarrow \frac{x^2 - 4}{x+1} \cdot \frac{1}{x-2}$$

$$\frac{\overset{1}{(x+2)(\cancel{x-2})}}{(x+1)(\cancel{x-2})} \rightarrow \frac{x+2}{x+1}$$

Name: _____

Date: _____

Class: _____

Mixed Expressions and Complex Fractions Worksheet

Simplify.

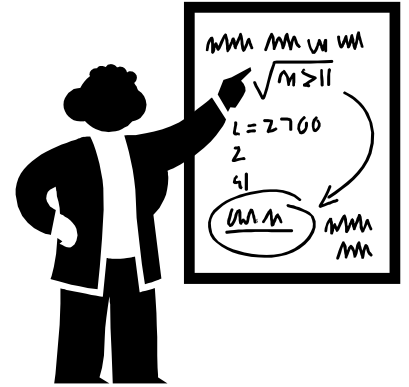
1. $\frac{3\frac{1}{2}}{4\frac{3}{4}}$

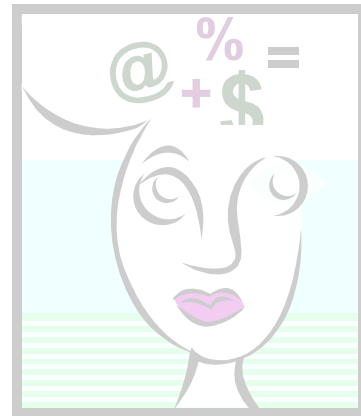
2. $\frac{\frac{x^2}{y}}{\frac{y}{x^2}}$

3. $\frac{\frac{x+4}{y-2}}{\frac{x^2}{y^3}}$

4. $\frac{\frac{x^3}{y^2}}{\frac{x+y}{x-y}}$

5. $\frac{\frac{x+y}{a+b}}{\frac{x^2-y^2}{a^2-b^2}}$





$$6. \frac{\frac{x^2 + 8x + 15}{x^2 + x - 6}}{\frac{x^2 + 2x - 15}{x^2 - 2x - 3}}$$

$$7. \frac{\frac{a^2 - 2a - 3}{a^2 - 1}}{a - 3}$$

$$8. \frac{x - \frac{15}{x-2}}{x - \frac{20}{x-1}}$$

$$9. \frac{x + 2 + \frac{2}{x+5}}{x + 6 + \frac{6}{x+1}}$$

$$10. \frac{x + 5 + \frac{3}{x+1}}{x - 1 - \frac{3}{x+1}}$$

Mixed Expressions and Complex Fractions Worksheet Key

Simplify.

$$1. \frac{3\frac{1}{2}}{4\frac{3}{4}}$$

$$\frac{\frac{7}{2}}{\frac{19}{4}} \rightarrow \frac{7}{2} \cdot \frac{4}{19} \rightarrow \frac{7}{\cancel{2}} \cdot \frac{\cancel{4}^2}{19} \rightarrow \frac{14}{19}$$

$$2. \frac{\frac{x^2}{y}}{\frac{y}{x^2}}$$

$$\frac{x^2}{y} \cdot \frac{x^2}{y} \rightarrow \frac{x^2}{y} \cdot \frac{x^2}{y} \rightarrow \frac{x^4}{y^2}$$

$$3. \frac{\frac{x+4}{y-2}}{\frac{x^2}{y^3}}$$

$$\frac{x+4}{y-2} \cdot \frac{y^3}{x^2} \rightarrow \frac{y^3(x+4)}{x^2(y-2)}$$

$$4. \frac{\frac{x^3}{y^2}}{\frac{x+y}{x-y}}$$

$$\frac{x^3}{y^2} \cdot \frac{x-y}{x+y} \rightarrow \frac{x^3(x-y)}{y^2(x+y)}$$

$$5. \frac{\frac{x+y}{a+b}}{\frac{x^2-y^2}{a^2-b^2}}$$

$$\frac{1(x+y)}{1(a+b)} \cdot \frac{(a+b)(a-b)}{(x+y)(x-y)} \rightarrow \frac{\cancel{1(x+y)}^1 \cdot \cancel{(a+b)}^1 (a-b)}{\cancel{1(a+b)}_1 \cdot \cancel{(x+y)}_1 (x-y)} \rightarrow \frac{a-b}{x-y}$$

$$6. \frac{\frac{x^2 + 8x + 15}{x^2 + x - 6}}{\frac{x^2 + 2x - 15}{x^2 - 2x - 3}}$$

$$\frac{(x+3)(x+5)}{(x+3)(x-2)} \rightarrow \frac{(x+3)(x+5)}{(x+5)(x-3)} \cdot \frac{(x+1)(x-3)}{(x+3)(x-2)} \rightarrow$$

$$\frac{\overset{1}{\cancel{(x+3)}} \overset{1}{(x+5)}}{\overset{1}{(x+3)}(x-2)} \cdot \frac{\overset{1}{(x+1)} \cancel{(x-3)}}{\cancel{(x+5)}(x-3)} \rightarrow \frac{x+1}{x-2}$$

$$7. \frac{\frac{a^2 - 2a - 3}{a^2 - 1}}{a - 3}$$

$$\frac{(a-3)(a+1)}{(a+1)(a-1)} \rightarrow \frac{(a-3)(a+1)}{1(a-3)} \cdot \frac{1}{1(a-3)} \rightarrow$$

$$\frac{\overset{1}{\cancel{(a-3)}} \overset{1}{(a+1)}}{\overset{1}{(a+1)}(a-1)} \cdot \frac{1}{1\cancel{(a-3)}} \rightarrow \frac{1}{a-1}$$

$$8. \frac{x - \frac{15}{x-2}}{x - \frac{20}{x-1}}$$

$$\frac{\frac{x(x-2)-15}{x-2}}{\frac{x(x-1)-20}{x-1}} \rightarrow \frac{\frac{x^2-2x-15}{x-2}}{\frac{x^2-x-20}{x-1}} \rightarrow \frac{\frac{(x-5)(x+3)}{1(x-2)}}{\frac{(x-5)(x+4)}{1(x-1)}} \rightarrow$$

$$\frac{(x-5)(x+3)}{1(x-2)} \cdot \frac{1(x-1)}{(x-5)(x+4)} \rightarrow$$

$$\frac{\cancel{(x-5)}(x+3)}{1(x-2)} \cdot \frac{1(x-1)}{\cancel{(x-5)}(x+4)} \rightarrow \frac{(x+3)(x-1)}{(x-2)(x+4)}$$

$$9. \frac{x+2+\frac{2}{x+5}}{x+6+\frac{6}{x+1}}$$

$$\frac{(x+2)(x+5)+2}{x+5} \rightarrow \frac{x^2+7x+12}{x^2+7x+12} \rightarrow \frac{(x+3)(x+4)}{1(x+5)} \rightarrow$$

$$\frac{(x+3)(x+4)}{1(x+5)} \cdot \frac{1(x+1)}{(x+3)(x+4)} \rightarrow$$

$$\frac{\cancel{1} \quad \cancel{1}}{1(x+5)} \cdot \frac{1(x+1)}{\cancel{(x+3)}\cancel{(x+4)}} \rightarrow \frac{x+1}{x+5}$$

$$10. \frac{x+5+\frac{3}{x+1}}{x-1-\frac{3}{x+1}}$$

$$\frac{(x+5)(x+1)+3}{x+1} \rightarrow \frac{x^2+6x+8}{x^2-4} \rightarrow \frac{x^2+6x+8}{x+1} \cdot \frac{x+1}{x^2-4} \rightarrow \frac{(x+4)(x+2)}{1(x+1)} \cdot \frac{1(x+1)}{(x+2)(x-2)} \rightarrow$$

$$\frac{(x+4)\cancel{(x+2)}}{1\cancel{(x+1)}} \cdot \frac{1\cancel{(x+1)}}{\cancel{(x+2)}(x-2)} \rightarrow \frac{x+4}{x-2}$$

Student Name: _____

Date: _____

Mixed Expressions and Complex Fractions Checklist

1. On question 1, did the student simplify the expression correctly?
 - a. Yes (20 points)
 - b. Student wrote numerator and denominator as rational numbers but made errors when multiplying by the reciprocal of the denominator (15 points)
 - c. Student wrote numerator or denominator correctly as rational numbers (not both), then preceded to multiply by the reciprocal of the denominator (10 points)
 - d. Student wrote numerator and denominator incorrectly as rational numbers, then preceded to multiply by the reciprocal of the denominator (5 points)

2. On question 2, did the student simplify the expression correctly?
 - a. Yes (20 points)
 - b. Student wrote numerator and denominator as rational numbers but made errors when multiplying by the reciprocal of the denominator (15 points)
 - c. Student wrote numerator or denominator correctly as rational numbers (not both), then preceded to multiply by the reciprocal of the denominator (10 points)
 - d. Student wrote numerator and denominator incorrectly as rational numbers, then preceded to multiply by the reciprocal of the denominator (5 points)

3. On question 3, did the student simplify the expression correctly?
 - a. Yes (20 points)
 - b. Student wrote numerator and denominator as rational numbers but made errors when multiplying by the reciprocal of the denominator (15 points)
 - c. Student wrote numerator or denominator correctly as rational numbers (not both), then preceded to multiply by the reciprocal of the denominator (10 points)
 - d. Student wrote numerator and denominator incorrectly as rational numbers, then preceded to multiply by the reciprocal of the denominator (5 points)

4. On question 4, did the student simplify the expression correctly?
 - a. Yes (20 points)
 - b. Student wrote numerator and denominator as rational numbers but made errors when multiplying by the reciprocal of the denominator (15 points)
 - c. Student wrote numerator or denominator correctly as rational numbers (not both), then preceded to multiply by the reciprocal of the denominator (10 points)
 - d. Student wrote numerator and denominator incorrectly as rational numbers, then preceded to multiply by the reciprocal of the denominator (5 points)

5. On question 5, did the student simplify the expression correctly?
 - a. Yes (20 points)
 - b. Student wrote numerator and denominator as rational numbers but made errors when multiplying by the reciprocal of the denominator (15 points)
 - c. Student wrote numerator or denominator correctly as rational numbers (not both), then proceeded to multiply by the reciprocal of the denominator (10 points)
 - d. Student wrote numerator and denominator incorrectly as rational numbers, then proceeded to multiply by the reciprocal of the denominator (5 points)

6. On question 6, did the student simplify the expression correctly?
 - a. Yes (20 points)
 - b. Student wrote numerator and denominator as rational numbers but made errors when multiplying by the reciprocal of the denominator (15 points)
 - c. Student wrote numerator or denominator correctly as rational numbers (not both), then proceeded to multiply by the reciprocal of the denominator (10 points)
 - d. Student wrote numerator and denominator incorrectly as rational numbers, then proceeded to multiply by the reciprocal of the denominator (5 points)

7. On question 7, did the student simplify the expression correctly?
 - a. Yes (20 points)
 - b. Student wrote numerator and denominator as rational numbers but made errors when multiplying by the reciprocal of the denominator (15 points)
 - c. Student wrote numerator or denominator correctly as rational numbers (not both), then proceeded to multiply by the reciprocal of the denominator (10 points)
 - d. Student wrote numerator and denominator incorrectly as rational numbers, then proceeded to multiply by the reciprocal of the denominator (5 points)

8. On question 8, did the student simplify the expression correctly?
 - a. Yes (20 points)
 - b. Student wrote numerator and denominator as rational numbers but made errors when multiplying by the reciprocal of the denominator (15 points)
 - c. Student wrote numerator or denominator correctly as rational numbers (not both), then proceeded to multiply by the reciprocal of the denominator (10 points)
 - d. Student wrote numerator and denominator incorrectly as rational numbers, then proceeded to multiply by the reciprocal of the denominator (5 points)

9. On question 9, did the student simplify the expression correctly?
- a. Yes (20 points)
 - b. Student wrote numerator and denominator as rational numbers but made errors when multiplying by the reciprocal of the denominator (15 points)
 - c. Student wrote numerator or denominator correctly as rational numbers (not both), then preceded to multiply by the reciprocal of the denominator (10 points)
 - d. Student wrote numerator and denominator incorrectly as rational numbers, then preceded to multiply by the reciprocal of the denominator (5 points)

10. On question 10, did the student simplify the expression correctly?
- a. Yes (20 points)
 - b. Student wrote numerator and denominator as rational numbers but made errors when multiplying by the reciprocal of the denominator (15 points)
 - c. Student wrote numerator or denominator correctly as rational numbers (not both), then preceded to multiply by the reciprocal of the denominator (10 points)
 - d. Student wrote numerator and denominator incorrectly as rational numbers, then preceded to multiply by the reciprocal of the denominator (5 points)

Total Number of Points _____

- A 180 points and above
- B 160 points and above
- C 140 points and above
- D 130 points and above
- F 129 points and below

**Any score below C
needs
remediation!**